$$= \underbrace{(\bar{\Omega}_t - \Phi_t}_{=-\kappa_t} + \underbrace{\Phi_t - \Omega_{t-1})}_{=\lambda_t} \underbrace{\Omega_{t-1}^{-1}\ \xi_{t-1}}_{=\mu_{t-1}} + \underbrace{\Omega_{t-1}\ \Omega_{t-1}^{-1}}_{=I}\ \xi_{t-1} + \bar{\Omega}_t\ F_x^T\ \delta_t$$

$$= \xi_{t-1} + (\lambda_t - \kappa_t)\ \mu_{t-1} + \bar{\Omega}_t\ F_x^T\ \delta_t$$

Since λ_t and κ_t are both sparse, the product $(\lambda_t - \kappa_t)\ \mu_{t-1}$ only contains finitely many non-zero elements and can be calculated in constant time. Further, $F_x^T\ \delta_t$ is a sparse matrix. The sparseness of the product $\bar{\Omega}_t\ F_x^T\ \delta_t$ follows now directly from the fact that $\bar{\Omega}_t$ is sparse as well.

12.4.2 Measurement Updates

The second important step of SLAM concerns the update of the filter in accordance to robot motion. The measurement update in SEIF directly implements the general extended information filter update, as stated in lines 6 and 7 of Table 3.6, page 76:

$$\Omega_t = \bar{\Omega}_t + H_t^T\ Q_t^{-1}\ H_t \tag{12.16}$$

$$\xi_t = \bar{\xi}_t + H_t^T\ Q_t^{-1}\ [z_t - h(\bar{\mu}_t) + H_t\ \mu_t] \tag{12.17}$$

Writing the prediction $\hat{z}_t = h(\bar{\mu}_t)$ and summing over all individual elements in the measurement vector leads to the form in lines 13 and 14 in Table 12.3:

$$\Omega_t = \bar{\Omega}_t + \sum_i H_t^{iT}\ Q_t^{-1}\ H_t^i \tag{12.18}$$

$$\xi_t = \bar{\xi}_t + \sum_i H_t^{iT}\ Q_t^{-1}\ [z_t^i - \hat{z}_t^i + H_t^i\ \mu_t] \tag{12.19}$$

Here Q_t, δ, q, and H_t^i are defined as before (e.g., Table 11.2 on page 348).

12.5 Sparsification

12.5.1 General Idea

The key step in SEIFs concerns the sparsification of the information matrix Ω_t. Because sparsification is so essential to SEIFs, let us first discuss it in general terms before we apply it to the information filter. Sparsification is an approximation through which a posterior distribution is approximated by two of its marginals. Suppose a, b, and c are sets of random variables (not to be confused with any other occurrence of these variables in this book!), and suppose we are given a joint distribution $p(a, b, c)$ over these variables. To sparsify this distribution, we have to remove any direct link between the