calculated as follows:

$$
\begin{aligned}
G_t^{-1} &= (I + F_x^T\ \Delta\ F_x)^{-1} \\
&= (I \underbrace{- F_x^T\ I\ F_x + F_x^T\ I\ F_x}_{=0} + F_x^T\ \Delta\ F_x)^{-1} \\
&= (I - F_x^T\ I\ F_x + F_x^T\ (I + \Delta)\ F_x)^{-1} \\
&= I - F_x^T\ I\ F_x + F_x^T\ (I + \Delta)^{-1}\ F_x \\
&= I + \underbrace{F_x^T\ [(I + \Delta)^{-1} - I]\ F_x}_{\Psi_t} \\
&= I + \Psi_t
\end{aligned}
\tag{12.12}
$$

By analogy, we get for the transpose $[G_t^T]^{-1} = (I + F_x^T\ \Delta^T\ F_x)^{-1} = I + \Psi_t^T$. Here the matrix $\Psi_t$ is only non-zero for elements that correspond to the robot pose. It is zero for all features in the map, and hence can be computed in constant time. This gives us for our desired matrix $\Phi_t$ the following expression:

$$
\begin{aligned}
\Phi_t &= (I + \Psi_t^T)\ \Omega_{t-1}\ (I + \Psi_t) \\
&= \Omega_{t-1} + \underbrace{\Psi_t^T\ \Omega_{t-1} + \Omega_{t-1}\ \Psi_t + \Psi_t^T\ \Omega_{t-1}\ \Psi_t}_{\lambda_t} \\
&= \Omega_{t-1} + \lambda_t
\end{aligned}
\tag{12.13}
$$

where $\Psi_t$ is zero except for the sub-matrix corresponding to the robot pose. Since $\Omega_{t-1}$ is sparse, $\lambda_t$ is zero except for a finite number of elements, which correspond to active map features and the robot pose.

Hence, $\Phi_t$ can be computed from $\Omega_{t-1}$ in constant time, assuming that $\Omega_{t-1}$ is sparse. Equations (12.11) through (12.13) are equivalent to lines 5 through 9 in Table 12.2, which proves the correctness of the information matrix update in **SEIF_motion_update**.

Finally, we show a similar result for the information vector. From (12.2) we obtain

$$
\bar{\mu}_t = \mu_{t-1} + F_x^T\ \delta_t \tag{12.14}
$$

This implies for the information vector:

$$
\begin{aligned}
\bar{\xi}_t &= \bar{\Omega}_t\ (\Omega_{t-1}^{-1}\ \xi_{t-1} + F_x^T\ \delta_t) \\
&= \bar{\Omega}_t\ \Omega_{t-1}^{-1}\ \xi_{t-1} + \bar{\Omega}_t\ F_x^T\ \delta_t \\
&= (\bar{\Omega}_t + \Omega_{t-1} - \Omega_{t-1} + \Phi_t - \Phi_t)\ \Omega_{t-1}^{-1}\ \xi_{t-1} + \bar{\Omega}_t\ F_x^T\ \delta_t \\
&= (\bar{\Omega}_t \underbrace{- \Phi_t + \Phi_t}_{=0} \underbrace{- \Omega_{t-1} + \Omega_{t-1}}_{=0})\ \Omega_{t-1}^{-1}\ \xi_{t-1} + \bar{\Omega}_t\ F_x^T\ \delta_t
\end{aligned}
\tag{12.15}
$$